

Preliminary results from simulations of temperature oscillations in Stirling engine regenerator matrices

Stig Kildegård Andersen^{a,*}, Henrik Carlsen^a, Per Grove Thomsen^b

^a*Department of Mechanical Engineering, Technical University of Denmark, DK-2800 Lyngby, Denmark*

^b*Informatics and Mathematical Modelling, Technical University of Denmark, DK-2800 Lyngby, Denmark*

Abstract

The objective of this study has been to create a Stirling engine model for studying the effects of regenerator matrix temperature oscillations on Stirling engine performance.

A one-dimensional model with axial discretisation of engine components has been formulated using the control volume method. The model contains a system of ordinary differential equations (ODEs) derived from mass and energy balances for gas filled control volumes and energy balances for regenerator matrix control masses. Interpolation methods with filtering properties are used for state variables at control volume interfaces to reduce numerical diffusion and/or non-physical oscillations. Loss mechanisms are included directly in the governing equations as terms in the mass and energy balances.

Steady state periodic solutions that satisfy cyclic boundary conditions and integral conditions are calculated using a custom built shooting method.

It has been found possible to accurately solve the stiff ODE system that describes the coupled thermodynamics of the gas and the regenerator matrix and to reliably find periodic steady state solutions to the model. Preliminary results indicate that the regenerator matrix temperature oscillations do have significant impact on the regenerator loss, the cycle power output, and the cycle efficiency and thus deserve further study.

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1. Introduction

Computer simulation has been used for design and optimisation of Stirling engines for several decades. Early modelling efforts resulted in very simple models that could be solved on the computers of

* Corresponding author. Tel.: +45 4525 4163.

E-mail address: ska@mek.dtu.dk (S.K. Andersen).

Nomenclature

A	wetted surface area [m^2].
c_p	spec. heat for const. pressure [$\text{J}/(\text{kg K})$]
c_v	spec. heat for const. volume [$\text{J}/(\text{kg K})$]
E	energy [J]
h	conv. heat transfer coeff. [$\text{W}/(\text{m}^2 \text{K})$]
m	mass [kg]
$\dot{m}$	mass flow [kg/s]
p	pressure [Pa]
t	time [s]
T	temperature [K]
V	volume [m^3]
cv	in control volume
lbnd	at left boundary of control volume
rbnd	at right boundary of control volume

the day. As computers grow ever more powerful the many assumptions in early models can be gradually relaxed in order to achieve a more accurate prediction of engine behaviour. With more accurate models new insight into phenomena occurring in engines can be gained and engine designs can be optimised to achieve higher thermal efficiencies.

One classical assumption in Stirling engine models is that the temperature of all metal in the engine remains constant during the cycle. In one of the key components in a Stirling engine, the regenerator, the validity of this assumption can be questionable. Analytical studies, such as the studies by Jones [1] suggest that metal temperature oscillations in regenerators can have an impact on engine performance.

Experimental studies of regenerators have mostly been conducted in order to generate correlations for heat transfer and flow friction, such as those compared by Thomas and Pittman [2], that can be used in simulation programs. Several experimental studies, for instance the work of Gedeon and Wood [3] and of Isshiki et al. [4], have been performed by placing small samples of regenerator matrix material into specialised regenerator test equipment instead of performing the measurements on complete regenerators inside actual engines. These studies have yielded information about the flow friction and heat transfer in regenerator matrices but they do not show the influence of regenerator performance on machine performance. In the heat transfer study by Siegel [5], measurements have been performed on a full regenerator inside an actual Stirling engine. But it has only been possible to measure bulk temperatures and pressure losses that result from the combined effects of all phenomena, such as heat transfer, compression/expansion, matrix temperature oscillations, and flow friction, occurring in the regenerator in the engine; The influence of the individual phenomena has not been revealed. Some authors report applying correction terms to simulation results calculated with constant regenerator matrix temperatures in order to account for the effects of regenerator matrix temperature oscillations, see for instance Jones or Kühl and Schultz [1,6].

This paper presents an overview of the method and the initial findings of an effort to explore the effects of dynamic temperature oscillations in regenerator matrices using a numerical simulation model.

2. The stirling engine

Similarly to an internal combustion engine the Stirling engine is based on a gas cycle, where work is expended to compress a cold working gas and where work is extracted by expanding the gas after it has been heated in order to increase the pressure. The Stirling engine, however, works on a closed gas cycle, where the working gas does not take part in combustion and is not exchanged in every cycle. Instead the heating and cooling of the working gas is achieved by sending the working gas back and forth through a serial connection of three heat exchangers, viz. a heater, a cooler, and a regenerator, configured as sketched in Fig. 1.

The efficiency of a Stirling engine can be improved by the regenerator because it can recycle some of the heat that is removed from the gas during transfer to the cold cylinder to preheat the gas when it is transferred back to the hot cylinder. The regenerator is usually made as a porous matrix with a large heat transfer area and a large specific heat capacity from, for instance, metal felt. The regenerator acts as a thermal heat storage that absorbs heat when hot gas flows through it and then releases it again when the flow direction is reversed and the gas flow becomes cooler than the matrix. At steady state periodic operating conditions there is a steep temperature gradient through the regenerator matrix dictated to some degree by the temperatures of the heater and cooler. In order to achieve a high thermal efficiency, it is very important to have a well performing regenerator so that the energy flux loss through the regenerator from the heater to the cooler is minimised. At the same time the pressure drop across the regenerator must be kept as low as possible in order to avoid wasting too much work on pushing the gas back and forth. Regenerator optimisation is thus at the heart of Stirling engine design.

Many practical Stirling engine designs look very different from Fig. 1 but the net effects of compression, expansion, and pushing gas back and forth through heat exchangers and a regenerator are the same.

The engine design parameters and operating conditions that are needed as input parameters for the Stirling engine model used in this study are based on an existing 9 kW engine design by Carlsen and Bovin [7]. A drawing of the region around the cylinder of this engine can be seen in Fig. 2. This engine uses one working piston and a displacer piston instead of the two working pistons in the engine of Fig. 1.

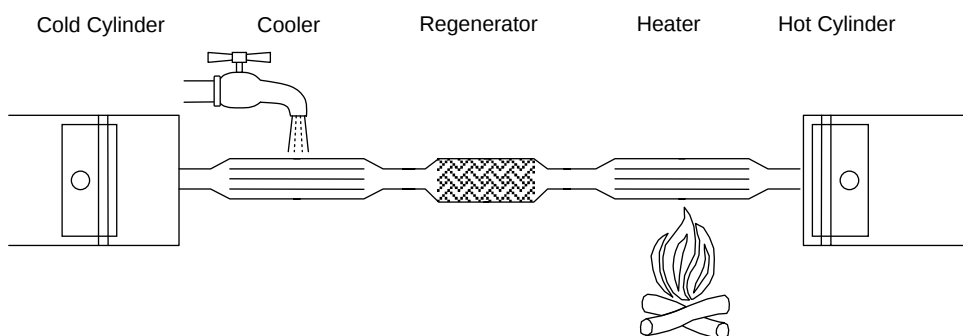


Fig. 1. Stirling engine with two heat exchangers and a regenerator.

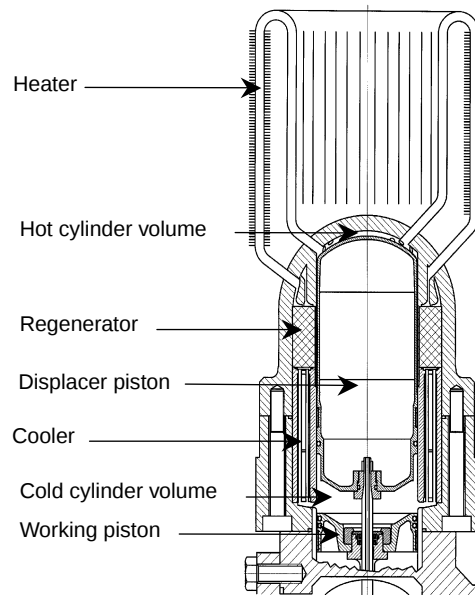


Fig. 2. Cylinder region of 9 kW Stirling engine.

3. The stirling engine model

The Stirling engine model has been formulated as a system of ODEs using the control volume method and conservation of mass and energy.

3.1. Discretisation in model

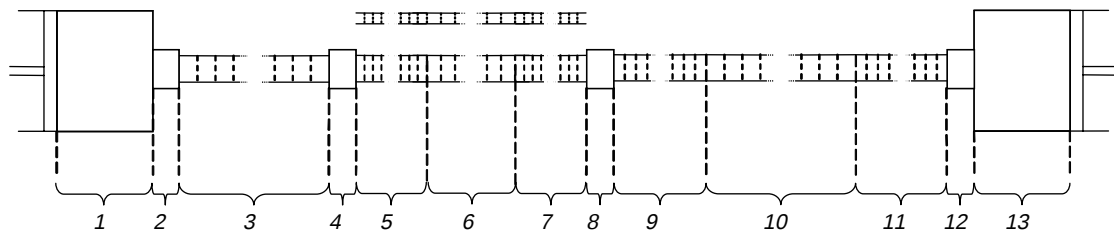
For the sake of simplicity the geometry of the model is not referred to Fig. 2 but to the equivalent geometry in Fig. 1.

The cylinder volumes and the manifold volumes between the components are represented by single control volumes. The sizes of the control volumes representing the cylinder volumes vary in a cyclic fashion.

In the model, the regenerator and the heater are split into three sub components each. In the case of the heater this division has been chosen because the end sections of the heater tubes do not receive the same external heat input as the central part of the tubes. Treating the inactive parts of the heater as separate sub components simplifies the model code. The regenerator has been split into three components to facilitate non-uniform spatial discretisation where finer grids are used in the ends of the regenerator than in the central part. This helps to better resolve localised phenomena in the ends of the regenerator with minimal computational efforts.

The cooler and the sub components in the regenerator and heater are further subdivided into parameterised numbers of control volumes.

The control volumes in the discretisation described above are linked together into a string of control volumes that represents the space in the engine occupied by the working gas.

**Components:**

1: Cold cylinder volume

3: Cooler

5, 6, 7: Regenerator

13: Hot cylinder volume

9, 10, 11: Heater

2, 4, 8, 12: Manifold volumes

Fig. 3. Discretisation used in Stirling engine model.

A string of control masses that represents the regenerator matrix has also been defined. This string of control masses only spans the length of the regenerator and uses the same axial discretisation as the corresponding control volumes for the working gas.

The resulting discretisation of the engine is illustrated in Fig. 3.

3.2. Phenomena included in model

The working gas is helium and is modelled as an ideal gas with non-constant thermophysical properties.

The model has been formulated using control volumes, control masses, and conservation of mass and energy. Conservation of momentum has been reduced by assuming that the axial inertia of the gas is negligible. More accurately, the velocities of gas flows between control volumes are found using steady state correlations for pressure losses in tube flow, through regenerator matrices, and through flow constraints.

Heat transfer is included between the gas and the wetted metal walls of all components. The instantaneous rates of heat exchange between gas and walls is calculated from the wall temperatures and the average temperatures in the gas filled control volumes using empirical correlations for heat exchange in tube flow, flow through porous matrices and flow in cylinder volumes. The temperatures of the walls in the cylinders, manifold volumes, the heater, and the cooler are assumed fixed at their cycle mean values.

In the regenerator heat transfer is included between the gas and the regenerator matrix. The temperatures of the regenerator matrix control masses can be treated in two different ways. Either the temperatures can be assumed fixed at their cycle mean values or they can be modelled using ODEs derived from an energy balance for a lumped control mass, i.e. a control mass with a uniform temperature. The validity of using a lumped formulation for the matrix thread material has been verified using a separate model that resolves the radial temperature variations inside a thread subjected to the conditions in the regenerator.

Heat conduction inside the metal walls of the engine is also included. This heat conduction affects the temperatures of the metal walls in contact with the gas and thus also the gas. The main heat conduction paths in the model are illustrated by the arrows in Fig. 4.

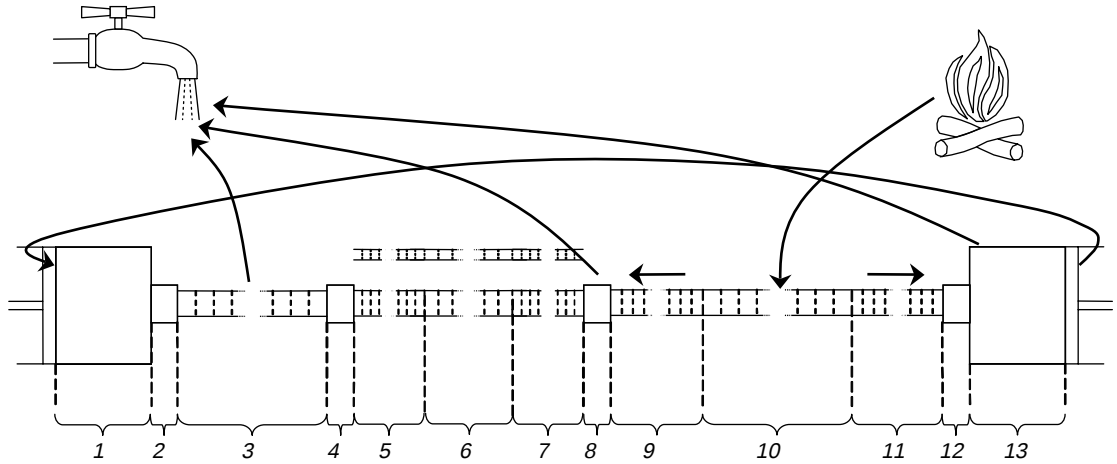


Fig. 4. Heat conduction paths in Stirling engine model.

3.3. Model equation system

The control volumes that represent the working gas are described by two ODEs each, viz. a mass balance and an energy balance. The mass balance ODEs take the form of Eq. (1).

$$\frac{dm_{cv}}{dt} = \dot{m}_{lbnd} - \dot{m}_{rbnd} \quad (1)$$

Flows across boundaries are represented as positive in the direction from left to right (from cold to hot) in Fig. 3. The influence of flow friction on the mass distribution in the engine is directly included in the mass balance equations because the mass flow rates are calculated from correlations for pressure losses.

In the energy balances for the control volumes, it has been assumed that kinetic and gravitational potential energies in the control volumes are of negligible magnitudes. The total energy in the control volumes has thus been assumed equal to the internal energy. Because of this the energy balances for the control volumes take the form of Eq. (2).

$$\frac{dE_{cv}}{dt} = \frac{d(m_{cv}c_vT_{cv})}{dt} = \dot{m}_{lbnd}c_pT_{lbnd} - \dot{m}_{rbnd}c_pT_{rbnd} + hA_{ht}(T_w - T_{cv}) - p_{cv}\frac{dV_{cv}}{dt} \quad (2)$$

The first two terms in Eq. (2) represent enthalpy flows between neighbouring control volumes. Notice that interpolated boundary temperatures are used in these terms to avoid numerical diffusion. The interpolation is carried out using interpolation methods with filtering properties similar to the mixed interpolation and extrapolation method presented in by Kühl and Schultz [8]. The third term represents convective heat exchange between gas and metal. The last term represents work from volume changes and is only relevant for the cylinder volumes. Energy loss terms such as metal heat conduction typically affect the wall temperatures in the engine and are thus included in the ODEs through the convective heat exchange term.

The control masses that represent the regenerator matrix are described either by ODEs of the form Eq. (2) with only a heat exchange term or by a constant temperature. Heat conduction through the metal in the matrix is not taken into account.

In order to be able to find the temperatures of metal walls throughout the engine it is also necessary to integrate the heat transfer to the wall sections during each cycle. ODEs that have just the heat exchange term of Eq. (2) are used for this purpose.

The governing equations in the model make up a coupled system of first order ODEs.

3.4. Mathematical problem definition

For the purposes of this investigation steady state periodic solutions to the ODE system of the model are needed. In a steady state periodic solution the values of all variables representing masses and energies must be the same at the end and the beginning of the cycle. This requirement is a cyclic boundary condition. The initial masses and energies in the control volumes and control masses must be found so that cyclic boundary conditions are satisfied.

Cyclic heat transfer is assumed to take place for the metal walls in the cylinders, manifold volumes, and in the end sections of the heater. When constant regenerator matrix temperatures are assumed then conditions for cyclic heat transfer are also used for finding the matrix temperatures.

Integrations of heat transfer to or from the wall sections and portions of the regenerator matrix, where cyclic heat transfer is assumed must result in zero net heat transfer if the prescribed constant temperatures are to be true steady state cycle mean values. The fixed wall and matrix temperatures must be found so that this is achieved.

An infinite number of solutions satisfy the above conditions for steady state solutions for the model investigated here. In order to get solutions that are directly comparable, it is necessary to add an additional condition that fixes the total mass of the working gas in the engine. For this purpose, an integral condition that measures the deviation from a user specified mean pressure in the hot cylinder has been chosen. The amount of mass and energy in the string of control volumes representing the working gas must be scaled to eliminate this deviation.

The mathematical problem that must be solved thus contains a cyclic boundary value problem for a system of ODEs, a set of integral conditions for cyclic heat transfer, and an integral condition that must be used for scaling the masses and energies in the gas filled control volumes. The numerical method must also allow additional integrations that can be used for integrating the heat absorbed in the heater and/or cooler and the work done on the pistons, so that performance characteristics, such as power output and thermal efficiency, can be calculated.

4. Implementation of model and numerical method

The mathematical problem of the model is solved numerically using the MusSim (Multi purpose software for Simulation) software by Andersen [9,10]. The software and the model is implemented in standard Fortran95 and has been tested to be compatible with Compaq, Intel, Lahey/Fujitsu, HP, IBM, and Sun compilers and thus runs on a large number of platforms.

A good Stirling engine model should approach periodic steady state asymptotically if integrated forward in time from a given set of initial values, but the convergence may be slow and asymptotical. To drastically accelerate this convergence the MusSim software uses a purpose built shooting method. This shooting method treats the ODEs in the model as a boundary value problem that is solved for initial values and parameters corresponding to a periodic steady state solution to the initial value problem (IVP)

defined by the ODEs. The cyclic boundary conditions and the integral conditions are treated as residual equations that must be solved in much the same way as a Newton–Raphson method solves non-linear residual equations. The main difference is that the shooting method requires some of the variables to be scaled based on a residual.

The system of ODEs turns out to be very stiff. This is not surprising as the variables in the system have very different inertial properties. For instance, the mass of gas and the energy in a control volume where gas is blowing through at high velocity can change on a much smaller time scale than the thermal energy stored in a control mass in the regenerator matrix. In order to handle this stiffness a suitable IVP method must be used in the shooting method. During this investigation the semi-implicit GERK method by Thomsen [11] has been successfully applied.

To be able to achieve accurate results it has proven necessary to solve ill conditioned non-linear equation systems with very strict tolerances in the internal iterations of the GERK method. This is handled using the built in non-linear equation solver of the MusSim software. It can be time consuming to robustly seek out difficult solutions of this kind but the MusSim software has features to speed up this process and this makes the model manageable for many purposes on a standard PC.

5. Model verification

A great deal of effort has been put into verifying the correctness of the model.

One of the most obvious and also most important tests is to make sure that the model does not leak mass and/or energy due to misconnected control volumes or control masses. Testing for conservation of mass is trivial and has been done by monitoring the sum of the masses of gas in the control volumes. Testing for conservation of energy in the same manner is slightly more involved as it requires the system to be kept adiabatic from the surroundings and it requires the work terms from the movement of the pistons to be zeroed out. For steady state solutions, though, conservation of energy can be verified by an overall energy balance applied at the outer boundaries of the model. These tests are integrated into the model and are performed for every calculated periodic steady state solution. Conservation of mass and energy has been found to satisfy relative tolerances that are stricter than the tolerance enforced by the GERK method for the individual masses and energies.

The correctness of the implementations of the correlations needed in the model have been tested by exchanging them with different correlations and checking the differences caused by the exchanges. The variations have been found to be within the expected ranges.

The calculated performance of the engine according to the assumed best model has been compared to values measured on the actual 9 kW engine. Depending on the combination of correlations used to describe pressure losses and heat transfer the model overestimates the work output from the gas cycle by 8–21% and the efficiency of the gas cycle by 6–18%. These results were expected as the model does not account for certain known losses associated with the displacer piston that divides the cylinder volume of the engine.

6. Preliminary results and discussion

After initial verification of the model had been carried out it was studied how the spatial discretisation in the regenerator influences the calculated temperature profile for the regenerator matrix. During this

investigation the cooler was divided into 10 control volumes and the total length of the heater was divided into 20 control volumes.

For fixed matrix temperatures the total length of the regenerator was divided into 10, 20, and 50 control volumes of uniform size in subsequent simulations. The resulting matrix temperature profiles are shown in Fig. 5. The slightly curved shape of the calculated temperature profiles is largely independent of the fineness of the discretisation.

This experiment was repeated with dynamic regenerator temperatures. In order to present the information from these simulations it has been chosen to plot the maximum and minimum energy based temperatures in the regenerator matrix control masses during a cycle. These plots are shown in Fig. 6 and it is apparent that the shapes of the profiles depend on the fineness of the discretisation. It also appears that the curvatures of the temperature profiles are largest in the ends of the regenerator. This large curvature in the ends of the matrix is caused by the matrix exchanging considerable amounts of energy with incoming gas flows whose temperatures differ significantly from the temperatures of the ends of the matrix.

Based on these observations a new non-uniform discretisation with a larger density of control volumes in the ends of the regenerator was devised. In this discretisation the ends of the regenerator (components 5 and 7 in Fig. 3) were both given lengths corresponding to 8% of the total length of the regenerator while component 6 covered the remaining central part. Component 5 was divided into 10 control volumes, component 6 into five control volumes, and the fineness of the discretisation in component 7, located where the largest gradients were observed in Fig. 6, was varied between 10 and 20 control volumes. Temperature profiles showing the maximum and minimum temperatures in the matrix control masses during the cycle when using the new discretisation are shown in Fig. 7. The temperature profiles in Fig. 7 appear very similar and because of this the discretisation has not been further refined.

Fig. 8 shows comparisons of the temperature profiles for fixed and dynamic matrix temperatures calculated using fine discretisations. The figure shows that the temperature profile in the regenerator matrix looks significantly different when the oscillations of the matrix temperatures are taken into account. The main differences are the temperature oscillations in the ends of matrix and a slightly flatter temperature gradient in the central part of the regenerator.

In order to verify that the solutions with dynamic matrix temperatures found by the shooting method of the MusSim software are true steady state solutions a test procedure was devised.

In this test procedure the initial values found by the shooting method are used as initial values for sequential simulations. These simulations run the engine for an additional 100 consecutive cycles from

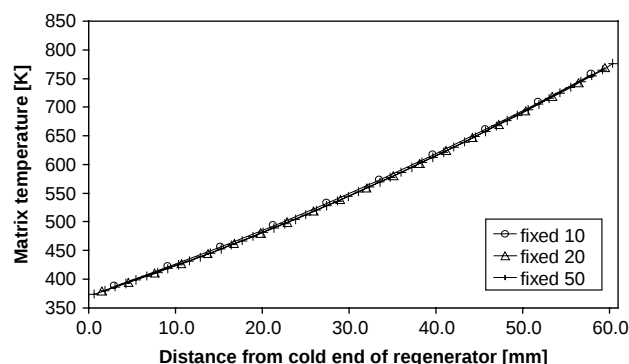


Fig. 5. Axial temperature profiles in the regenerator matrix calculated with fixed matrix temperatures.

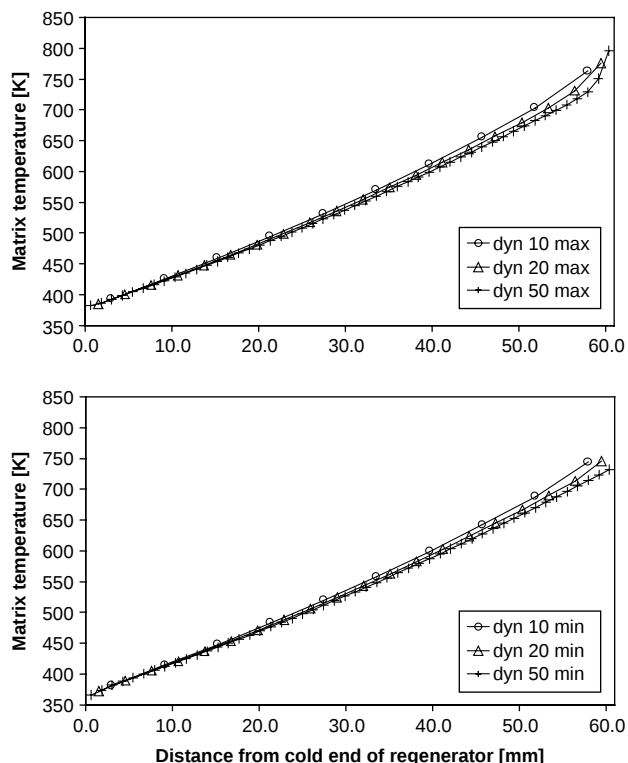


Fig. 6. Max. and min. matrix temperature profiles calculated with dynamic matrix temperatures and uniform discretisation.

the initial values found by the shooting method. After completing the 100 cycles the final values are recorded. The initial values found by the shooting method are then subtracted from these final values and the resulting differences are divided by the initial values. The results of this procedure we label as the relative drifts in the periodic steady state solution.

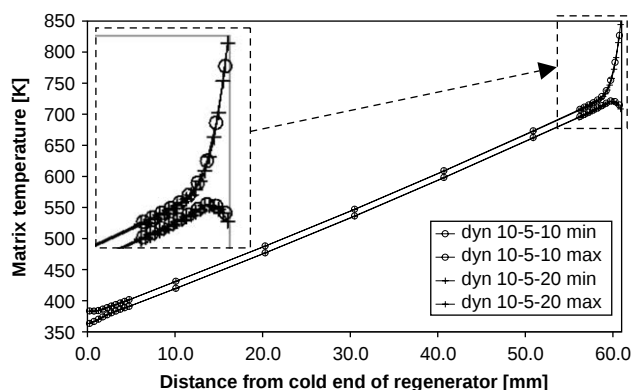


Fig. 7. Max. and min. temperature profiles in the regenerator matrix calculated with dynamic matrix temperatures and non-uniform discretisation.

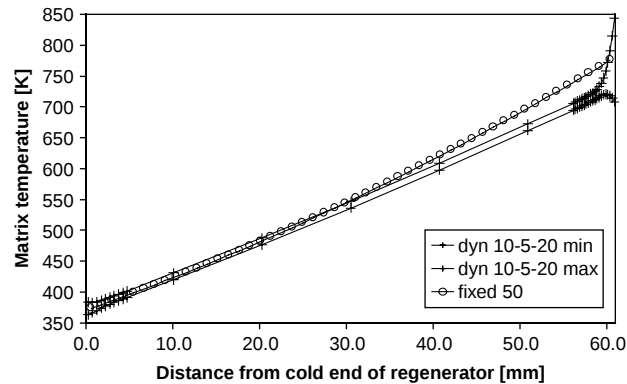


Fig. 8. Comparisons of temperature profiles for fixed and dynamic matrix temperatures.

This test procedure has been completed for the above mentioned five discretisations used in the simulations with dynamic regenerator temperatures. The calculated relative drifts for all variables are shown in Fig. 9 for the case, where shooting has been carried out to a tolerance of 10^{-5} using a relative tolerance of 10^{-6} in the GERK method. The figure shows that the relative drifts of all variables in all the tested solutions are below the tolerance enforced by the shooting method. The figure also shows that the signs of the largest drifts seem random in nature within components. They can thus be interpreted as noise from the integration of the ODEs. The variables most affected by this noise are the masses of gas in the regenerator control volumes and, to a lesser extent, the masses and energies in and close to the cylinder volumes.

The visual inspection of the temperature profiles presented above illustrates that the temperature profiles appear to be shaped differently when oscillations in matrix temperatures are taken into account. The visual inspection does not, however, show that the temperature oscillations have an impact on the performance of Stirling engines. The effect on performance by the temperature oscillations is shown by Table 1 that contains characteristic numbers for the performance of the gas cycle in the engine for all the solutions presented above. The values in the table are the work output, the heat intake from the heater, the thermal efficiency, and the regenerator loss defined as the average net energy flux through the regenerator from the hot end towards the cold end of the regenerator.

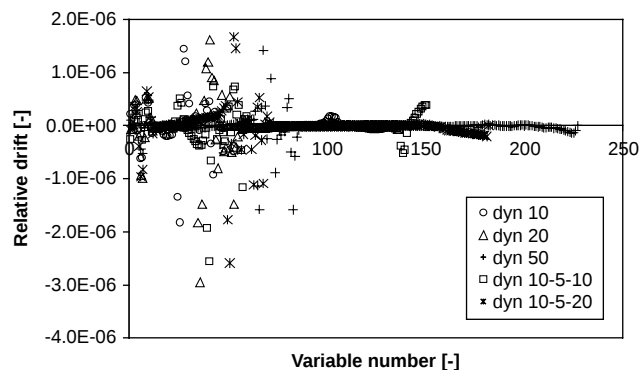


Fig. 9. Relative drifts in periodic steady state solutions.

Table 1
Calculated performance of gas cycle in engine

	Work (kW)	Heat input (kW)	Efficiency (%)	Reg. loss (kW)
Fixed 10	15.213	31.938	47.633	1.519
Fixed 20	15.192	31.886	47.647	1.519
Fixed 50	15.187	31.873	47.650	1.518
Dyn 10	14.617	30.738	47.554	0.543
Dyn 20	14.542	30.622	47.489	0.528
Dyn 50	14.392	30.478	47.222	0.496
Dyn 10-5-10	14.306	30.474	46.946	0.470
Dyn 10-5-20	14.285	30.456	46.904	0.467

The numbers in Table 1 show several distinct differences in performance caused by taking dynamic temperature oscillations in the regenerator matrix into account.

The net energy flux loss through the regenerator is reduced by roughly two thirds. In spite of the reduction of this loss term the thermal efficiency is reduced due to a significant reduction in the power output of the cycle. These differences will be further investigated.

When comparing the above performance numbers for the gas cycle of the engine to the performance of this and other engines it is important to remember that the above performance numbers are for the gas cycle of the engine and not for the shaft power from the engine or for the electrical power output from an engine-generator assembly.

7. Conclusion

It has been found that true steady state periodic solutions can be reliably calculated for a cyclic boundary value problem that describes a Stirling engine and includes the coupled thermodynamics of a gas and a regenerator matrix.

It has also been found that the calculated temperature profile in a regenerator matrix can look significantly different when matrix temperature oscillations are taken into account. The main differences have been found to be significant matrix temperature oscillations in the ends of the regenerator and a less steep matrix temperature gradient in the central part of the regenerator.

Finally, it has been found that the oscillations in the regenerator matrix temperatures influence the calculated performance of the gas cycle in the engine. The observed effects include a reduction of the regenerator loss, the power output, and the thermal efficiency.

Future work will include improvements to the model and the MusSim software as well as comparisons of obtained simulation results with measurements on a test bench.

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